

Ratio, Proportion and Variation- Key Points

Ratio:

1. Duplicate ratio of $a : b$ is $a^2 : b^2$
2. Sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$
3. Triplicate ratio of $a : b$ is $a^3 : b^3$
4. Sub-triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$

If $a/b = c/d$; then $a=bc/d$

(i) $(a+b)/b = (c+d)/d$

(ii) $(a-b)/b = (c-d)/d$

(iii) $a/c = b/d$

(iv) $b/a = d/c$

(v) $(a+b)/(a-b) = (c+d)/(c-d)$

If $a/b=c/d$; then simplest possible value of $a=c$, $b=d$

(vi) In the ratio $a : b$, if $a > b$, then $(a + x / b + x) < a / b$ ($x > 0$)

(vii) In the ratio $a : b$, if $a < b$, then $(a + x / b + x) > a / b$ ($x > 0$)

(viii) In the ratio $a : b$, if $a = b$, then $(a + x / b + x) = a / b$ ($x > 0$)

Proportion & Variation:

If a is directly proportional to b ; then $a=kb$

If a is inversely proportional to b ; then $a=k/b$

If a is directly proportional to b and inversely proportional to c , then a is directly proportional to b/c

$\Rightarrow a= kb/c$

Exercise Questions

1. Ram, Sham and Suresh start business investing in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{6}$. The time for which each of them invested their money was in the ratio 8:6:12 respectively. If they get profit of Rs.18000 from the business, then how much share of profit will Ram get?

- a. Rs.4000
- b. Rs.6000
- c. Rs.8000
- d. Rs. 10000

2. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

- a. 8 : 9
- b. 17 : 18
- c. 21 : 22
- d. Cannot be determined

3. p,q and r are three positive numbers and $Q = \frac{p+q+r}{2}$; If $(Q-p):(Q-q):(Q-r) = 2:5:7$, then find the ratio of p,q and r ?

- a. 4:3:7
- b. 12:9:7
- c.9:7:4
- d. 4:3:2

4. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?

- a. Rs. 460
- b. Rs. 484
- c. Rs. 550
- d. Rs. 664

5. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is

- a. 2 : 5
- b. 3 : 5
- c. 4 : 5
- d. 6 : 7

6. The ratio of the cost prices of two articles A and B is 4:5. The articles are sold at a profit with their selling prices being in the ratio 5:6. If the profit on article A is half of its cost price, find the ratio of the profits on the articles A and B?

- a. 7:10
- b. 9:11
- c. 5: 9
- d. 10:11

7. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is the total amount?

- a. Rs. 14000
- b. Rs. 15000
- c. Rs. 20000
- d. None of these

8. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2}:2/3:3/4$, then the first part is:

- a. Rs. 182
- b. Rs. 190
- c. Rs. 196
- d. Rs. 204

9. A bag contains 50 paise, 20 paise and 10 paise coins in the ratio 5:3:1. If the total amount in the bag is 640 Rs, find the difference in the amounts contributed by 50 paise and 20 paise coins.

- a. Rs.300
- b. Rs.400
- c. Rs.380
- d. None of these

10. The speed of an engine is proportional to the square root of the number of wagons attached to it. Without any wagons attached to it the speed of the engine is 60km/hr. With 16 wagons attached to it the speed of the engine is 40km/hr; find the maximum number of wagons that can be attached so that the train moves.

- a. 144
- b. 145
- c. 142
- d. 143

Answer Key:

1.c; 2.c; 3.b; 4.b; 5.c; 6.d; 7.a; 8.d; 9.c; 10.d

Concepts and Theory

Ratio Proportion and Variation

Ratio: The ratio $a : b$ represents a fraction a/b . a is called antecedent and b is called consequent. Ratio is the relation between two numbers which is expressed by a fraction. The equality of two different ratios is called proportion.

If $a : b = c : d$ then a, b, c, d are in proportion. This is represented by $a : b :: c : d$.

In $a : b = c : d$, then we have $a * d = b * c$.

If $a/b = c/d$ then $(a + b) / (a - b) = (d + c) / (d - c)$. The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as $a : b$. Ratio of two quantities is always an abstract number (without any units).

In the ratio $a : b$, we call ' a ' as the first term or antecedent and ' b ', the second term or consequent.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

Tips: If A is thrice as good a workman as B , then Ratio of work done by A and $B = 3 : 1$.
Ratio of times taken by A and B to finish a work = $1 : 3$

General formula can be extended if more than 2 people (or machines are working together)
 $1/TA + 1/TB + 1/TC + \dots = 1/T_{\text{together}}$; where TA, TB and TC are the times taken by A, B and C respectively to complete the task alone and T_{together} is the time taken by them to complete the task when they are all working together.

Eg: If Alex can build a house in 2 days and his apprentice Bob can build a house in 3 days, then how long will it take Alex and Bob to build a house when they are working together?

Putting the information from the question into the formula gives us,
Invert both sides of the equation $\text{Time working together} = 6/5 = 11/5$ days. So Alex and Bob will take $11/5$ days to build a house when they are working together.

Proportion: The equality of two ratios is called proportion. If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.
Product of means = Product of extremes. Thus, $a : b :: c : d = (b \times c) = (a \times d)$.

1. If $a : b :: c : d$, then $a + b : b :: c + d : d$
2. If $a : b :: c : d$, then $a - b : b :: c - d : d$

3. If $a : b :: c : d$, then $a + b : a - b :: c + d : c - d$
 4. If $a/b = c/d = e/f = \dots k$, then $k = (a \pm c \pm e \dots) / (b \pm d \pm f \dots)$

- Tips:** 1) Direct proportion: If x is directly proportional to y : $x_1/y_1 = x_2/y_2$
 2) Indirect proportion: If x is inversely proportional to y : $x_1 y_1 = x_2 y_2$

Variation:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write,
 $x \propto y$

Chain Rule

Direct Proportion: Two quantities are said to be proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.

- Eg. 1) Cost is directly proportional to the number of articles
 (More articles, More Cost)
 Eg. 2) Work done is directly proportional to the number of men work (More Men, More Work)

Indirect proportion : Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.

- Eg. 1) The time taken by a car is covering a certain distance is inversely proportional to the speed of the car. (More speed, Less is the time taken to cover a distance)
 Eg. 2) The time taken to finish a work is inversely proportional to the number of persons working at it. (More persons, Less is the time taken to finish a job)

Remarks: In solving problems by chain rule, we compare every item with the term to be found out.

Key Notes:

If $p:q :: r:s$
 $\Rightarrow s = qr/p$
 The method of finding the 4th term of a proportion when three are given is known as rule of three as above.
 Three or more quantities are said to be in compound proportion if one quantity depends on the other remaining quantities.

If p,q,r,s are four quantities & if $p:q :: r:s$, then

1) Componendo
 $(p+q)/q = (r+s)/s$

2) Dividendo
 $(p-q)/p = (r-s)/s$

3) Componendo & Dividendo
 $(p+q)/(p-q) = (r+s)/(r-s)$

4) Invertendo
 $q/p = s/r$

5) Alternendo
 $p/r = q/s$

Direct proportion is indicated by arrows in the same direction, Inverse proportion is indicated by arrows in opposite direction.

Exercise Questions

1. If $A : B = 4 : 7$ and $B : C = 5 : 9$ then $A : B : C$ is :

a) 20 : 35 : 63 b) 35 : 36 : 63 c) 30 : 35 : 65 d) 25 : 34 : 68

2. If $m : n = 3 : 4$ and $n : o = 8 : 9$, then $m : o$ is :

a) 1 : 3 b) 3 : 2 c) 2 : 3 d) 1 : 2

3. If 15% of A is the same as 20% of B, then $A : B$ is :

a) 3 : 4 b) 4 : 3 c) 17 : 16 d) 16 : 17

4. Which of the following ratios is greatest ?

- a) 7 :15 b) 15 :23 c) 17 :25 d) 21 :29

5. The ratio of number of boys and girls in a class is 3 : 2. In the 1st semester exam 20% of boys and 25% of girls get more than or equal to 90% marks. What percentage of students get less than 90% marks ?

- a) 56 b) 70 c) 78 d) 80

6. A mixture of 85 kg contains milk and water in the ratio 27 : 7. How much more water is to be added to get a new mixture containing milk and water in the ratio 3:1?

- a) 5 kg b)6.5 kg c) 7.5 kg d) 8 kg

7. The ages of Raju and Biju are in the ratio 3:1. Fifteen years hence, the ratio will be 2:1. Their present ages are:

- a) 30yrs, 10yrs b) 45 yrs, 15yrs c)21 yrs, 7 yrs d) 60yrs, 20yrs

8. The speeds of three motor bikes are in the ratio 6 : 5 : 4. The ratio between the time taken by them to travel the same distance is :

- a) 10 : 12 : 15 b) 12 : 10 : 8 c) 15 : 12: 10 d) 10 : 15 : 12

9. In a company 10% of male staff are same in number as 1/4th of the female staff. What is the ratio of male staff to female staff

- a) 3 : 2 b) 5 : 2 c) 2 : 1 d) 4 : 3

10. The telephone bill of a certain establishment is partly fixed and partly varies as the number of calls consumed. When in a certain month 540 calls made the bill is Rs.1800. In another month 620 calls are

consumed then the bill becomes Rs.2040. In another month 500 units are consumed due to more holidays. The bill for that month would be :

- a) Rs.1560 b) Rs.1680 c) 1840 d) Rs.1950

11. If 0.4 of a number is equal to 0.06 of another number, the ratio of the numbers is :

- a) 2 : 3 b) 3 : 4 c) 3 : 20 d) 20 : 3

12. The ratio of incomes of two person P1 and P2 is 5 : 4 and the ratio of their expenditures is 3 : 2. If at the end of the year, each saves Rs.1600, then what is the income of P1?

- a)Rs.800 b) Rs.2400 c)Rs.4000 d)3200

13. The mean proportional between 234 and 104 is :

- a) 12 b) 39 c) 54 d) None of these

14. The seats in an Engineering college for Computer science, electronics and civil are in the ratio of 5 : 7 :

8. There is a proportion to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats ?

- a) 2 : 3 : 4 b) 6 : 7 : 8 c) 6 : 8 : 9 d) none of these

15. If 96 is divided into four parts proportional to 5, 7, 4, 8 then the smallest part is :

- a) 16 b)14 c)20 d) 18

16. The Price of 357 apples is Rs.2499. What will be the price of 49 dozens of such apples?

- a) Rs.3800 b) Rs.2816 c)Rs.4116 d) Rs.3116

17. Some ladies can do a piece of work in 12 days. Two times the number of such ladies will do half of that

work in :

- a) 6 days b) 4 days c) 12 days d) 3 days

18. A piece of work can finish by a certain number of men in 100 days. If however, there were 10 men less,

it would take 10 days more for the work to be finished. How many men were there originally?

- a) 75 b) 82 c) 100 d) 110

19. It takes 10 days for digging a trench of 100 m long, 50 m broad and 10 m deep. What length of trench,

25 m broad and 15 m deep can be dug in 30 days ?

- a) 400 m b) 200 m c) 100 m d) 89m

20. A wheel rotates 10 times every minute and moves 20 cm during each rotation. How many metres does

the wheel move in one hour?

- a) 6 metre b) 12 metre c) 120 metre d) 1200 metre

Answer & Explanations

1. Expl : $A : B = 4 : 7$

$$B : C = 5 : 9 = 5 \times 7/5 : 9 \times 7/5 = 7 : 63/5$$

$$A : B : C = 4 : 7 : 63/5 = 20 : 35 : 63$$

2. Expl : $m/n = m/n \times n/o = 3/4 \times 8/9 = 2/3$. So $m : o = 2 : 3$

3. Expl : $15\% \text{ of } A = 20\% \text{ of } B = 15A/100 = 20B/100 = 4/3 = 4:3$

4. Expl : $7/15 = 0.466$, $15/23 = 0.652$, $17/25 = 0.68$ $21/29 = 0.724$

Clearly, 0.724 is greatest and therefore, 21 : 29 is greatest

5. Expl : Let boys = 3x and girls = 2x.

Number of those who get less than 90% mark = (80% of 3x) + (75% of 2x)

$$= (80/100) * 3x + (75/100 * 2x) = 39x/10$$

Required percentage = $(39x/10 * 1/5x * 100)\% = 78\%$.

6. Expl : Milk in 85 kg of mix = $85 * (27/34) = 135/2$ kg.

Water in it = $85 - 135/2 = 35/2$ kg.

Let x kg of water be added to it.

Then, $(135/2)/(35/2 + x) = 3/1$; $135/(35 + 2x) = 3/1$;

$$105 + 6x = 135 ; 6x = 30; x = 5$$

The quantity of water to be added = 5 kg

7. Expl : Let the ages of Raju and Biju be 3x and x years respectively.

Then, $(3x + 15)/(x + 15) = 2/1$; $\rightarrow 2x + 30 = 3x + 15 \rightarrow x = 15$

So Raju's age = $3 * 15 = 45$ and Biju's age = 15 years

8. Expl : Ratio of time taken : $1/6 : 1/5 : 1/4 = 10 : 12 : 15$

9. Expl : 10% of MS = $1/4^{\text{th}}$ of FS $\rightarrow 10MS/100 = 1/4FS \rightarrow MS = 5/2 FS$

$$\therefore MS/FS = 5/2 = MS : FS = 5 : 2$$

10. Expl : Let the fixed amount be Rs. X and the cost of each unit be Rs. Y.

Then, $540y + x = 1800$ And $620y + x = 2040$

On subtracting (i) from (ii), we get $80y = 240 \rightarrow y = 3$

Putting $y = 3$ in (i) we get :

$$540 * 3 + x = 1800 \quad x = (1800 - 1620) = 180$$

\therefore Fixed charges = Rs.180, Charge per unit = Rs.3.

Total charges for consuming 500 units = $180 + (500 * 3) = \text{Rs.}1680$

11. Expl : $0.4A = 0.06B \rightarrow A/B = 0.06/0.40 = 6/40 = 3/20$

$$\therefore A : B = 3 : 20$$

12. Expl : Let the income of P1 and P2 be Rs. $5x$ and Rs. $4x$ respectively and let their expenditures be Rs. $3y$ and $2y$ respectively.

$$\text{Then, } 5x - 3y = 1600 \dots(i) \text{ and } 4x - 2y = 1600 \dots\dots(ii)$$

On multiplying (i) by 2, (ii) by 3 and subtracting, we get : $2x = 1600 \rightarrow x = 800$

$$P1's \text{ income} = Rs \ 5 * 800 = Rs.4000$$

13. Expl : Required mean proportion is = $\sqrt{234} \sqrt{104} = \sqrt{(13*9*2*13*18)}$
 $= (13*3*4) = 156$

14. Expl : Originally, let the number of seats for Computer science, electronics and civil are $5x : 7x : 8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$)

$$7x : 21x/2 : 14x = 14x : 21x : 28x = 2 : 3 : 4.$$

15. Expl : Given ratio = $5 : 7 : 4 : 8$, sum of ratio = 24

$$\therefore \text{The smallest part} = (96 * 4/24) = 16$$

16. Expl : More apples, more cost(Direct)

$$357 * 588 :: 2499 : x$$

$$x = (588 * 2499) / 357 = 4116$$

17. Expl : Let x ladies can do the work in 12 days. More ladies, less days (Indirect)

Less work, less days (direct)

$$\text{Ladies } 2x : x \quad \text{Work} \quad 1 : \frac{1}{2}$$

$$2x : x, 1 : \frac{1}{2} :: 12 : y$$

$$\therefore 2x * 1 * y = x * \frac{1}{2} * 12 \text{ or } y = 3$$

Hence the required number of days = 3

18. Expl: Originally let there be x men.

Less men, more days (Indirect)

$$\therefore (x-10) : x :: 100 : 110 \text{ or } x-10/x = 100/110$$

$$\text{or } 11x-110 = 10x \text{ or } x = 110$$

So, originally there were 110 men.

19. Expl : More days, more length (Direct)

Less breadth, more length (Indirect)

More depth, less length (Indirect)

Days 10 : 30;

Breadth 25 : 50; :: 100 : x

Depth 15 : 10;

$$\therefore 10 * 25 * 15 * x = 30 * 50 * 10 * 100$$

$$x = (30 * 50 * 10 * 100) / 10 * 25 * 15 = 400$$

So the required length = 400m

20. Expl : Number of times wheel moves in 1 hour = $10 * 60 = 600$

$$\therefore \text{Distance moves} = (600 * 20) \text{ cms} = 12000 \text{ cms}$$

In metres = 120 metre